Design of Compact Coupled-Line Couplers

Jacob C. Coetzee, *Member, IEEE* , James D. Cordwell and Shauna L. Waite

***Abstract*— The size of rat-race and branch-line couplers can be reduced by using periodic loading or artificial transmission lines. The objective of this work is to extend the idea of size reduction through periodic loading to coupled-line 90° hybrids. A procedure for the extraction of the characteristic parameters of a coupled-line 4-port from a single set of S-parameters is described. This method can be employed to design of coupled artificial transmission line couplers of arbitrary geometry. The procedure is illustrated through the design a broadside-coupled stripline hybrid, periodically loaded with stubs. Measured results for a prototype coupler confirm the validity of the theory.**

***Index Terms*— directional couplers, coupled lines, compact size, periodic structures, parameter extraction**

I. Introduction

THERE is a growing demand for reduced size components for applications in radio frequency or microwave systems. A number of researchers have proposed techniques for the miniaturization of directional couplers. Options include the use of space-filling curves to realize rat-race, branch-line and coupled-line hybrids [1]-[2]. Other techniques involve the use of lumped elements [3]-[4] or combinations of shunt capacitors and high-impedance transmission line sections [5]-[6]. An alternative approach involves the use of periodic loading or artificial transmission lines, which effectively decreases the physical length of 90° or 270° transmission line sections required in branch-line and rat-race couplers [7]-[8].

When lumped elements or periodic loading with stubs are employed to implement a transmission line section with an electrical length φ, the line is usually constructed using a predetermined number of unit cells. The first step in the design of a unit cell shown in Fig. 1(a) entails the analytical, numerical or experimental characterization of the structure. The parameters, component values or dimensions of the cell are determined by ensuring that they produce the required characteristic impedance, Z_0, and that the phase shift across each cell is a chosen fraction of φ. Equivalently, when numerical modeling of the unit cell is used, the parameters and dimensions are varied until $|S_{11}| = 0$ and $|\arg(S_{21})| = \varphi/N$, with N being the total number of unit cells connected in tandem to implement the transmission line section.

The objective of this work is to extend the idea of size reduction through periodic loading to coupled-line hybrid couplers. The modeling procedure for unit cells of single-line unit cells is not applicable to 4-port coupled lines, shown in Fig. 1(b). There is no direct relation between the S-parameters of the unit cell and those of the final coupler. For an optimized unit cell with an electrical length smaller than 90°, $|S_{31}| \neq 0$ and $|S_{41}| \neq k$, with k being the coupling coefficient of the final coupler. Instead, the even- and odd mode impedances and phase lengths of the coupled line section need to be extracted from the scattering parameters. Using these characteristic parameters as a guide, the physical dimensions of the unit cell can be adjusted to yield the desired response.

A procedure for the extraction of characteristic parameters of coupled lines was presented in [9]. This method requires a range of S-parameter samples at different frequencies. Another approach requires two sets of data: even/odd mode S-parameters computed with perfect magnetic/electric conductors on the plane of symmetry [10-11]. This method thus requires additional simulations and will be difficult to implement in cases where experimental data is to be used to characterize the coupled lines.

This paper presents a method for the extraction of the characteristic parameters from a single set of calculated or measured S-parameters. The method is applicable to coupled-line sections of any length and can be employed for the design of coupled artificial transmission line hybrids of arbitrary geometry. The procedure is illustrated through the design of a broadside-coupled hybrid, periodically loaded with stubs. Measured results for a prototype coupler are presented and confirm the validity of the theory.

II. Unit Element Modeling

A. Analysis of a general coupled-line section

Consider a section of coupled transmission lines, shown in Fig. 2. The even- and odd modes have distinct characteristic impedances, Z_{0e} and Z_{0o}. For a non-TEM structure, the phase velocities for the two modes are also different. For a specific physical length l, the section will have an electrical length of

$$\theta_{e,o} = \omega\sqrt{\varepsilon_{e,o}}\, l / c\,, \tag{1}$$

where ε_e and ε_o are the effective dielectric constants of the even and odd modes, respectively.

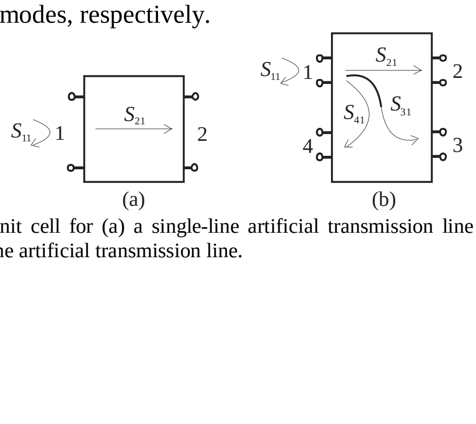

Fig. 1. Unit cell for (a) a single-line artificial transmission line and (b) a coupled-line artificial transmission line.

Manuscript received May 10, 2011.

The authors are with the School of Engineering Systems, Queensland University of Technology, GPO Box 2434, Brisbane, QLD 4001, Australia (e-mail: jacob.coetzee@qut.edu.au).

The coupled line section may be characterized using the approach described in [12], whereby the scattering matrix is obtained directly from knowledge of its eigenvectors and eigenvalues. The orthonormal eigenvectors of the scattering matrix $\mathbf{S}$ are found by inspection as $\mathbf{e}_1$ = ½$[1,-1,-1,1]^T$, $\mathbf{e}_2$ = ½$[1,1,1,1]^T$, $\mathbf{e}_3$ = ½$[1,1,-1,-1]^T$ and $\mathbf{e}_4$ = ½$[1,-1,1,-1]^T$. The corresponding eigenvalues are obtained by means of flow graphs for the reduced eigennetworks [12]. However, here the more general non-TEM case is considered by substituting the electrical length θ with θ_e or θ_o where appropriate, giving

$$\begin{aligned}\lambda_{1,2} &= S_{11} \mp S_{21} \mp S_{31} + S_{41} = \frac{\Gamma_e \mp e^{-j\theta_e}}{1 \mp \Gamma_e e^{-j\theta_e}} \\ \lambda_{3,4} &= S_{11} \pm S_{21} \mp S_{31} - S_{41} = \frac{\Gamma_o \pm e^{-j\theta_o}}{1 \pm \Gamma_o e^{-j\theta_o}}\end{aligned}, \qquad (2)$$

where $\Gamma_{e,o} = (Z_{0e,o} - Z_0)/(Z_{0e,o} + Z_0)$ and Z_0 is the selected system impedance. The scattering parameters are then obtained from

$$\begin{aligned}S_{11} &= \tfrac{1}{4}(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) \\ S_{21} &= \tfrac{1}{4}(-\lambda_1 + \lambda_2 + \lambda_3 - \lambda_4) \\ S_{31} &= \tfrac{1}{4}(-\lambda_1 + \lambda_2 - \lambda_3 + \lambda_4) \\ S_{41} &= \tfrac{1}{4}(\lambda_1 + \lambda_2 - \lambda_3 - \lambda_4)\end{aligned}. \qquad (3)$$

B. Extraction of characteristic parameters

One approach for extracting Z_{0e}, Z_{0o}, θ_e and θ_o from a given sample of scattering parameters at a specific frequency, involves the numerical solution of the real and imaginary components of the expressions in (2). An alternative method involves the calculation of the scattering parameters for one-half of the network with a magnetic wall (even-mode excitation) and an electric wall (even-mode excitation) on the plane of symmetry [10, 11]. The resulting 2-port networks will have even- and odd mode scattering matrices of

$$\mathbf{S}_{e,o} = \begin{bmatrix} S_{11e,o} & S_{21e,o} \\ S_{21e,o} & S_{11e,o} \end{bmatrix}. \qquad (4)$$

The corresponding impedance matrices can by calculated from

$$\mathbf{Z}_{e,o} = Z_0(\mathbf{I} + \mathbf{S}_{e,o})(\mathbf{I} - \mathbf{S}_{e,o})^{-1}. \qquad (5)$$

From [11], the even- and odd mode impedance matrix for a lossless coupled line is given by

$$\mathbf{Z}_{e,o} = j\begin{bmatrix} X_{11e,o} & X_{21e,o} \\ X_{21e,o} & X_{11e,o} \end{bmatrix}, \qquad (6)$$

where $X_{11e,o} = Z_{0e,o}\cot\theta_{e,o}$ and $X_{21e,o} = -Z_{0e,o}/\sin\theta_{e,o}$. The characteristic impedances and electrical lengths for both modes are thus obtained from

$$Z_{0e,o} = \sqrt{X_{21e,o}^2 - X_{11e,o}^2}, \quad \theta_{e,o} = \cos^{-1}(X_{11e,o}/X_{21e,o}). \qquad (7)$$

From [10], it follows that

$$S_{11} = \tfrac{1}{2}(S_{11e} + S_{11o}), \; S_{21} = \tfrac{1}{2}(S_{21e} + S_{21o}). \qquad (8)$$

Substituting (2) into (3) and comparing the even- and odd mode terms with those in (8) gives

$$\begin{aligned}S_{11e} &= \tfrac{1}{2}(\lambda_1 + \lambda_2) = S_{11} + S_{41}, \; S_{11o} = \tfrac{1}{2}(\lambda_3 + \lambda_4) = S_{11} - S_{41} \\ S_{21e} &= \tfrac{1}{2}(\lambda_2 - \lambda_1) = S_{21} + S_{31}, \; S_{21o} = \tfrac{1}{2}(\lambda_3 - \lambda_4) = S_{21} - S_{31}\end{aligned}. \qquad (9)$$

It is therefore not necessary to analyze both the even- and odd mode structures as in [10,11]. The scattering parameters of the full coupled line section can be employed to directly compute the characteristic modal impedances and electrical lengths from equations (9), (4), (5) and (7). The effective dielectric constants of the even- and odd modes are obtained from (1).

Consider the example listed in [9] of a coupled-line microstrip section with a physical length of l = 36.595 mm on RT/duroid 5880 substrate (ε_r = 2.2) with a thickness of 0.7874 mm. The scattering parameters at 1.5 GHz are listed as $S_{11} = -37.806$ dB $\angle -19.895°$, $S_{21} = -0.074$ dB $\angle -92.065°$, $S_{31} = -27.833$ dB $\angle 173.035°$, $S_{41} = -20.088$ dB $\angle -0.216°$. This single frequency sample was used to extract the characteristic even and odd mode parameters. There is close correlation between the results shown in the first column of Table I and those reported in reference [9], which required additional data at other frequencies for the calculation of the effective permittivities.

TABLE I
EXTRACTED CHARACTERISTIC PARAMETERS OF THE COUPLED-LINE SECTION

Current method	Reference [9]
$Z_{0e} = 55.951\,\Omega$	$Z_{0e} = 55.962\,\Omega$
$Z_{0o} = 45.826\,\Omega$	$Z_{0o} = 45.827\,\Omega$
$\varepsilon_e = 2.052$	$\varepsilon_e = 2.056$
$\varepsilon_o = 1.853$	$\varepsilon_o = 1.856$

C. Unit cell design

The design procedure for a coupled artificial transmission line hybrids involves an iterative process to optimize the dimensions of each unit cell. The scattering parameters of a unit cell with a chosen basic geometry are first determined for a range of dimensions and other geometrical parameters. The characteristic parameters Z_{0e}, Z_{0o}, ε_e and ε_o can be extracted for each sample. All sets of dimensions which satisfy $Z_{0e}\,Z_{0o} \approx Z_0^2$ are identified. From these, the set which yields the required coupling coefficient of $k=(Z_{0e}-Z_{0o})/(Z_{0e}+Z_{0o})$ is selected. Some fine-tuning to the length of the unit cell may be required to adjust its average electrical length until ½$(\theta_e+\theta_o)=(90/N)°$. The design is completed by connecting N unit cells in tandem to realize a coupled-line 90° hybrid coupler.

III. 5. DESIGN EXAMPLE

To illustrate the procedure, a broadside-coupled stripline coupler periodically loaded with stubs was designed. It was designed to have a coupling coefficient of k = −3.01 dB at the centre frequency of 1.8 GHz. The basic geometry of a unit cell is depicted in Fig. 3. The chosen substrate material is RT/duroid 5880 with relative permittivity ε_r=2.2 and a thickness of t_1=0.127 mm for the centre layer and t_2=0.7874 mm for the upper and lower layers.

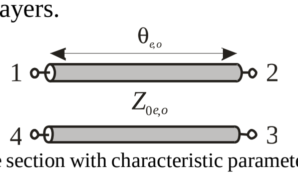

Fig. 2. A coupled-line section with characteristic parameters Z_{0e}, Z_{0o}, θ_e and θ_o.

It was decided to use N=4 unit cells to construct the coupler. The length of the unit cell was initially fixed to l_1=5 mm. Simulations were then carried out using CST Microwave Studio [13] for a range of dimensions with $3.43 \leq l_2 \leq 4.06$ mm, $0.152 \leq w_1 \leq 0.406$ mm and $0.559 \leq w_2 \leq 0.813$ mm. All sets of dimensions which approximately satisfy $Z_{0e} Z_{0o}$=50 Ω were identified. From those, the combination which best approximates k=0.7071 was found to be l_2=3.73 mm, w_1=0.305 mm and w_2=0.66 mm. The length of the unit cell was adjusted to l_1=4.683 mm so that $\theta_e+\theta_o$=45°. A photograph of the prototype coupler is shown in Fig. 4. The total length of the coupler circuit is 18.73 mm as opposed to a corresponding length of 28.07 mm for a conventional broadside-coupled stripline coupler.

There is good agreement between the calculated and measured amplitude response shown in Fig. 5. The phase balance between the through and coupled ports is shown in Fig. 6. The coupler displays good overall performance. It exhibits an amplitude balance of ±1 dB over the frequency range of 1.2–2.37 GHz and directivity of more than 16 dB. The measured phase difference between S_{21} and S_{41} is −90°±2° for 0.5–2.33 GHz.

IV. CONCLUSION

The procedure for the extraction of the characteristic parameters of coupled-line structures from a single sample of scattering parameters is simple to implement. Measured results for a broadside-coupled stripline hybrid periodically loaded with stubs confirm the validity of this approach. The prototype coupler is approximately 33% shorter than a conventional broadside-coupled stripline coupler, yet produces superior frequency performance. The principles described in this paper can be applied to coupled-line sections of any length and arbitrary geometry for the miniaturization of 90° hybrid couplers.

REFERENCES

[1] H. Ghali and T.A. Moselhy, "Miniaturized fractal rat-race, branch-line, and coupled-line hybrids", *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 11, pp. 2513- 2520, Nov. 2004.

[2] W.-L. Chen and G.-M. Wang, "Design of novel miniaturized fractal-shaped branch-line couplers", in *Proc. Asia-Pacific Microw. Conf. Proc.*, Dec. 2007, pp. 1-3.

[3] R. W. Vogel, “Analysis and design of lumped- and lumped- distributed element directional couplers for MIC and MMIC applications”, *IEEE Trans. Microw. Theory Tech.*, vol. 40, no. 2, pp. 253–262, Feb. 1992.

[4] T. Hirota, A. Minakaw, and M. Muraguchi, “Reduced-size branchline and rat-race hybrids for uniplanar MMIC’s”, *IEEE Trans. Microw. Theory Tech.*, vol. 38, no. 3, pp. 270–275, Mar. 1990.

[5] R. B. Singh and T. M. Weller, “Miniaturized 20 GHz CPW quadrature coupler using capacitive loading,” *Microw. Opt. Technol. Lett.*, vol. 30, no. 1, pp. 3–5, Jul. 2001.

[6] S.-S. Liao and J.-T. Peng, "Compact planar microstrip branch-line couplers using the quasi-lumped elements approach with nonsymmetrical and symmetrical T-shaped structure", *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 9, pp. 3508-3514, Sept. 2006.

[7] K.W. Eccleston and S.H.M. Ong, "Compact planar microstripline branch-line and rat-race couplers", *IEEE Trans. Microw. Theory Tech.*, vol. 51, no. 10, pp. 2119- 2125, Oct. 2003.

[8] C.-W. Tang and M.-G. Chen, "Synthesizing microstrip branch-line couplers with predetermined compact size and bandwidth", *IEEE Trans. Microw. Theory Tech.*, vol. 55, no. 9, pp. 1926-1934, Sept. 2007.

[9] M. Kang, *et. al* , “A new extraction method of characteristic parameters of a coupled transmission line”, in *Proc IEEE EPEP 2009*, Portland, Oct. 2009, pp. 187-190.

[10] R. Mongia, I. Bahl and P. Bhartia, *RF and Microwave Coupled Line Circuits*. Norwood, MA: Artech House, 1999, ch. 4.

[11] T. Fujii, Y. Kokubo, and I. Ohta, "High directivity quarter-wave microstrip couplers with periodic floating-conductors on coupled edges," in *Proc. 36th European Microwave Conference*, Manchester, Sept. 2006, pp. 32-35.

[12] W.J. Louw, “Coupled transmission lines revisited”, *IEEE Trans. Education*, vol.31, no.3, pp. 234-236, Aug 1988.

[13] CST Studio Suite 2010, Computer Simulation Technology AG, Darmstadt, Germany.

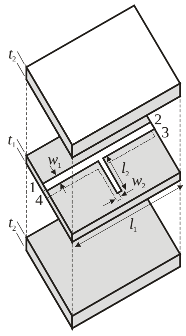

Fig. 3. Exploded view of a unit cell for a broadside-coupled stripline hybrid with coupled stubs.

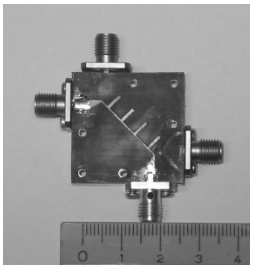

Fig. 4. Top view of the centre layer of the prototype coupler.

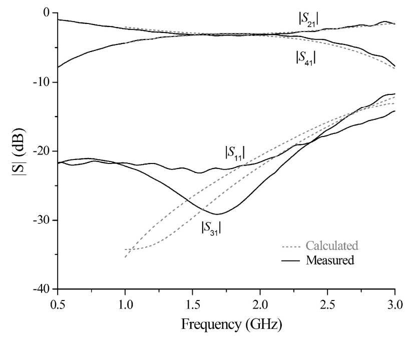

Fig. 5. Calculated and measured amplitude response of the prototype coupler.

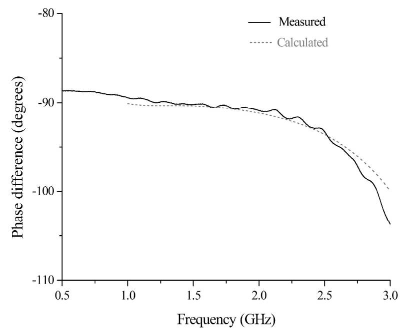

Fig. 6. Calculated and measured phase balance of the prototype coupler.